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A SYSTEM MEAN VOID FRACTION MODEL FOR PREDICTING VARIOUS TRANSIENT PHENOMENA ASSOCIATED WITH TWO-PHASE EVAPORATING AND CONDENSING FLOWS

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Abstract—The system mean void fraction model's principle virtue is its simplicity. The model converts the two-phase evaporating or condensing flow system into a type of lumped parameter system, generally yielding simple, closed form solutions in terms of the important system parameters. The particular applications of the model which are demonstrated in this paper are for a class of transient flow problems where complete vaporization or condensation takes place, and where the system mean void fraction can be considered to be time-invariant. This assumption uncouples the problem from the transient form of the momentum principle, an analytical simplification of considerable magnitude. The specific transients under consideration are caused by changes in the inlet flowrate. For evaporating flows, these transients are the effective liquid dry-out point, and the outlet flowrate of superheated vapor. For condensing flows, they are the effective point of complete condensation, and the outlet flowrate of subcooled liquid.

INTRODUCTION

The ability to predict the transient responses of two-phase evaporating or condensing flow systems is of considerable importance as it relates to both system design and control, whether the system is associated with nuclear or conventional power generation, refrigeration, or chemical processing.

Evaporating flow systems

For evaporating flow systems, some of the past efforts include Hudson, Atit & Bankoff's (1964) studies of the void fraction response to changes in power input and inlet flowrate, Zuber & Staub's (1966) investigation of void fraction propagation and wave form under oscillatory conditions, and Hancox & Nicoll's (1971) predictions of void fraction distributions in non-steady situations including heat flux modulation. Additional efforts include Gonzalez-Santalo & Lahey's (1973) investigation of local flow and quality responses for time varying inlet flowrates, and St. Pierre's (1965) study of the frequency response of void fraction to sinusoidal power modulation.

Essentially, in each of the above mentioned investigations, only partial evaporation was considered to take place in the evaporator; thus the flow quality leaving the system was less than unity. With respect to the specific transient phenomena under consideration in this paper, these references however, have only limited implications; in part because they deal with flow systems involving only partial evaporation, and in part because they consider different transient flow phenomena than what is presently under consideration.

Condensing flow systems

Because of the two-phase flow similarities in both processes, it seems reasonable to expect some similarities to exist between corresponding transient and flow instability manifestations for evaporating and condensing flow. Various types of instabilities have been reported for condensing flow; Vild, Schubert & Snoke (1968) investigated condenser stability for a space power system. Soliman & Berenson (1970) studied flow stability in both horizontal and vertical

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condenser tubes, concentrating primarily on pressure oscillations. Doroshchuk & Frid (1969) reported the utilization of the outlet flowrate oscillations from a condenser as an oscillatory inlet source for a study of evaporator stability. Schoenberg (1966) studied the frequency response of the inlet pressure and the point of complete condensation to sinusoidally varying inlet flowrates for a gas cooled mercury condenser.

There appears to be no reported work which has been directed toward the particular transient flow phenomena under consideration in this paper. In fact, condensing flow transients have apparently received very little attention as compared to their evaporating flow counterparts.

Methods of analysis

The generally accepted methods of analysis for two-phase flow are extensions of those already known for single phase flows; namely the application of the momentum and conservation of mass and energy principles, incorporating various simplifying assumptions, in an effort to obtain governing equations which are mathematically tractable. Although there are a variety of sub-models, the four major types of two-phase flows models listed by Collier (1972) and Wallis (1969) are the *homogeneous flow model*, the *separated flow model*, the *drift-flux model*, and the *various flow pattern models*. These models vary in complexity, and in general, for transient evaporating or condensing flows, result in a system of coupled partial differential equations.

Normally, a prerequisite to the formulation of any theoretical model which describes a certain physical phenomenon, is an understanding of the various mechanisms involved. However, when a particular mechanism influencing the phenomenon is very complex, or is not completely understood, it is sometimes possible to formulate a simplified model of the phenomenon which lumps the mechanism's effects into a single, determinable parameter. Such simplifications usually render the model more amenable to analysis, although they often place definite restrictions on the accuracy of the predictive results of the model. Therefore, the success of any theoretical model must be ultimately judged by its ability to predict the behaviour of the phenomenon, and to provide insight into the relative influence of the various physical and system parameters associated with the phenomenon.

The *system mean void fraction model* has been formulated in the spirit of the above philosophy. A variety of complicated flow patterns normally exist simultaneously in most two-phase evaporating or condensing flow processes; especially if complete vaporization or condensation takes place. Therefore, the complexity of the physical mechanisms involved is obvious, and a formulation of the transient form of the momentum principle for such processes is equally complex.

The *system mean void fraction model* is simpler than any of the four types of two-phase flow models mentioned above, in fact, its simplicity is its principle virtue. This model results in ordinary rather than partial differential equations. Obviously, the simplifications involved in the present form of the model place certain restrictions on its applicability; the major restrictions being the inability of the model to either predict or take into account specific localized phenomena. However, the model contains no empirical constants as such, and converts the two-phase evaporating or condensing flow system into a type of lumped parameter system, with an interesting range of transient flow applications as will be demonstrated.

SYSTEM MEAN VOID FRACTION MODEL

Certain aspects of this model were initially presented by Wedekind & Stoecker (1968). However, at that time, because various implications of the system mean void fraction were not as fully understood, it was not envisioned that the concept had the potential capability of being extended to include flow transients for condensing as well as evaporating flow. Therefore, the concept as originally presented was somewhat obscured by the fact that it merely represented a simplifying assumption in the formulation of a theoretical model which was amenable to solution. At this point in time however, with increasing experimental evidence of its generality to two-phase evaporating and condensing flows, a more formal presentation of the model seems warranted.

Various types of void fractions

In an effort to add both physical clarity and perspective to the model and its applications, a brief discussion of void fraction, with specific reference to its various levels of detail, is in order.

Local void fraction. Considering the continuously varying flow patterns which exist in two-phase liquid–gas flows, as well as the stochastic nature of the flow process, from a macroscopic Eulerian perspective, at an arbitrary point within the flow channel, an observer would detect discrete phase changes from one instant of time to the next. For example, the instantaneous local void fraction, $\alpha = \alpha(r, \theta, z, t)$, would be a discrete valued function of time, intermittently taking on values of zero or unity depending upon the particular phase which was instantaneously present.

Volumetric mean void fraction. The volumetric mean void fraction, α_v, represents the instantaneous value of the local void fraction, α, averaged over a given sampling volume, V; thus

$$\alpha_v = \alpha_v(t) = \frac{1}{V}\int_V \alpha(r, \theta, z, t)\,\mathrm{d}V. \qquad [1]$$

Most void fraction detectors sample a finite volume, even though for some detectors the sampling volume may be quite small. Therefore, for a finite volume, the volumetric mean void fraction loses much of the discreteness of the local void fraction and becomes more of a continuous function of time. However, because of the stochastic nature of the two-phase flow, it will still exhibit amplitude fluctuations. Jones & Zuber (1975) have investigated the statistical characteristics of these void fraction fluctuations as they relate to various flow patterns.

Area mean void fraction. The area mean void fraction, α_a, represents the instantaneous value of the local void fraction, α, averaged over the cross-sectional area of the flow channel, A; thus

$$\alpha_a = \alpha_a(z, t) = \frac{1}{A}\int_A \alpha(r, \theta, z, t)\,\mathrm{d}A. \qquad [2]$$

Like the volumetric mean void fraction, the area mean void fraction will also exhibit amplitude fluctuations as a consequence of the stochastic nature of the two-phase flow process. This is the most common type of void fraction found in the literature, common because it is ideally suited for one-dimensional analyses.

System mean void fraction

The system mean void fraction, α_s, will be defined here as representing the instantaneous value of the local void fraction averaged over the entire two-phase region under consideration; thus it is a special case of the volumetric mean void fraction. It can also be expressed in terms of the area mean void fraction averaged over the length, ζ, of the two-phase flow channel under consideration; thus

$$\alpha_s = \alpha_s(t) \equiv \frac{1}{\zeta}\int_{z=0}^{\zeta} \alpha_a(z, t)\,\mathrm{d}z. \qquad [3]$$

This definition assumes that $z = 0$ represents the beginning of the two-phase region under consideration. Again it is recognized that even for what is traditionally accepted as steady state conditions, the system mean void fraction, α_s, will manifest amplitude fluctuations due to the stochastic nature of the two-phase flow.

Systems with complete vaporization or condensation. For the two-phase evaporating or condensing flow system where complete vaporization or condensation takes place, the effective length of the two-phase region, ζ, will be designated by the symbol, η. It is clear that if the system is undergoing a flow or a heat-flux transient, then the length of the two-phase region would be a function of time; thus $\eta = \eta(t)$. However, even for what would conventionally be considered steady state conditions, the stochastic nature of the two-phase evaporating or condensing flow process will cause random fluctuations in the length of the two-phase region, $\eta(t)$. These fluctuations have been measured experimentally by Wedekind (1971).

In an effort to separate out the random fluctuations from the deterministic transients, an approach similar to what is used for describing turbulent flow is used; that is, time averaged quantities where the averaging time is large enough to eliminate the stochastic fluctuations, but short enough not to interfere with the deterministic transients. It is important to note that the term *fluctuation* is in reference to random behavior only and does not preclude the existence of a deterministic oscillatory behavior such as might arise in response to a sinusoidal type inlet flowrate. Therefore, the symbols $\bar{\eta}(t)$ and $\bar{\alpha}_s(t)$ represent the non-fluctuating effective length of the two-phase region and the non-fluctuating system mean void fraction respectively;§ thus from [3],

$$\bar{\alpha}_s(t) \equiv \frac{1}{\bar{\eta}(t)} \int_{z=0}^{\bar{\eta}(t)} \bar{\alpha}_a(z, t)\,\mathrm{d}z\,. \qquad [4]$$

From this point on, only non-fluctuating quantities will be considered. Therefore, in subsequent references to these quantities, no distinction will be made.

Implications of time-invariance. The remaining part of this paper will be directed at the class of transient two-phase evaporating and condensing flow problems where complete vaporization or condensation takes place, and where the system mean void fraction, $\bar{\alpha}_s$, is *time-invariant* (independent of time). A condition sufficient for this assumption to be valid is that the area mean void fraction, $\bar{\alpha}_a(z, t)$, be expressible as a function of a single dimensionless variable, ξ; that is

$$\bar{\alpha}_a(z, t) = \bar{\alpha}_a(\xi)\,, \qquad [5]$$

where

$$\xi \equiv z/\bar{\eta}(t)\,. \qquad [6]$$

This means that the independent variables z and t must enter into the expression for the area mean void fraction in a particular manner. A common example of being able to express a function of two independent variables as a function of a single new variable, which is itself a specified function of the two original variables, is the type of similarity relationships encountered in boundary layer theory.

The possibilities of the existence of such a similarity relationship have been investigated by Wedekind (1965). The conclusions were, that although physically such a relationship could not be satisfied exactly, the utilization of the similarity relationship would not lead to unacceptable errors, even for such extreme conditions as a 20 per cent step change in the inlet flowrate of the evaporating or condensing fluid.¶ This conclusion was arrived at by considering worst case situations, and estimating an upper and lower bound to the similarity relationship.

§Although it is beyond the scope of this paper, the implications of the fluctuating component of the system mean void fraction to other observed stochastic two-phase evaporating flow phenomena has been investigated by Wedekind & Beck (1974).

¶Strictly speaking, the analysis was done for evaporating flow; however, the extension to condensing flow seems to be equally valid.

From a physical perspective, the time-invariance of the system mean void fraction during a particular flow transient requires a specific redistribution of the liquid and vapor within the two-phase region. The amount of redistribution depends upon the magnitude of the flow change, and the rate at which the redistribution takes place must be much greater than that of the flow transient. Because of the order of magnitudes of the liquid and vapor velocities involved in most applications, the redistribution mechanism can not be dependent solely on the liquid velocity; it would be much too slow. Therefore, the much higher vapor velocity must be involved somehow in the liquid/vapor redistribution process in order for it to be fast enough.

It is recognized that the above qualitative description is incomplete and thus not very satisfying; but such is the current understanding of the redistribution mechanism. However, as will be demonstrated in a later section, the validity of assuming that the system mean void fraction is invariant with time is clearly established by the fact that theoretical models utilizing the concept are capable of accurately predicting results which can be experimentally measured.

It is also important to recognize the physical implications of the system mean void fraction simplification. It implies that for the class of transient phenomena for which the time invariance is valid, the transient form of the momentum principle is not important, thus indicating that the transient phenomena are governed by thermal mechanisms, that is, heat transfer and the conservation of mass and energy principles.

What is needed from the momentum principle however, is sufficient information to evaluate the system mean void fraction. In terms of its definition, as per [4], an analytical determination of its magnitude requires a knowledge of the axial distribution of the non-fluctuating area mean void fraction, $\bar{\alpha}_a(z, t)$, over the length of the two-phase region, $\bar{\eta}$. Analytical models by Levy (1960), Fujie (1964) and Zivi (1964) yielding this information require the application of the steady state form of the momentum principle, which yield a relationship between the area mean void fraction and the mean flow quality. The only additional information required is an estimate of the axial distribution of heat flux over the length of the two-phase region. Examples of the analytical determination of the system mean void fraction for both evaporating and condensing flows are given in the Appendix.

To summarize then, the assumption that the system mean void fraction is time-invariant during a particular two-phase evaporating or condensing flow transient implies that the *transient* form of the momentum principle is unnecessary to the solution of the problem. Therefore, only the *steady state* form of the momentum principle is required. Considering the complexity of the two-phase flow process, this represents a simplification of considerable magnitude.

APPLICATION TO EVAPORATING FLOW TRANSIENTS

This section of the paper is directed at the application of the system mean void fraction model to predict the transient responses of two-phase evaporating flow systems where the transients are caused by changes in the system inlet flowrate. The particular class of evaporating flow system under consideration here is one where complete vaporization takes place, and where the system mean void fraction, $\bar{\alpha}_s$, is assumed to be invarient with time. The two specific system responses to be considered are those of the mixture–vapor transition point (effective liquid dry-out point), and those of the outlet flowrate of the superheated vapor, the second being dependent upon the first.

Introduction

A schematic of the evaporating flow system is shown in figure 1. In addition to the system mean void fraction assumption, it will be assumed that there is some effective position in the evaporator where the last of the liquid is evaporated. Therefore, effects of entrainment are considered negligible; meaning that the dimensionless vapor velocity must be below the critical value for the onset of entrainment as established by Steen & Wallis (1964). This type of

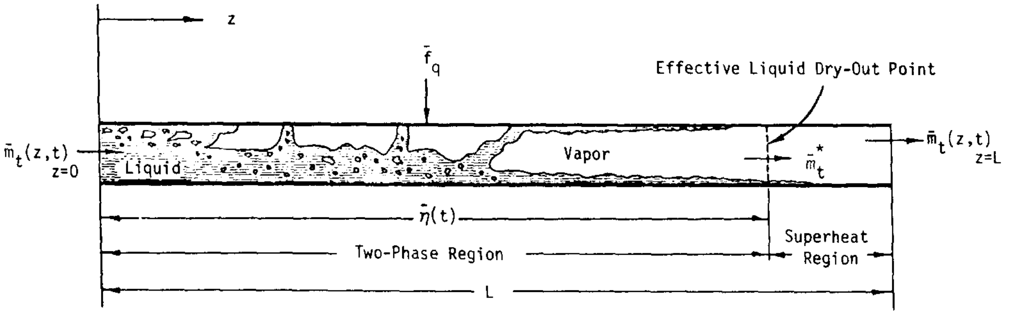

Figure 1. Schematic of horizontal evaporating flow system.

effective liquid dry-out point or mixture–vapor transition point has been observed and photographed by Wedekind & Stoecker (1968).

The evaporating system model will be formulated around the following simplifications:

1. Negligible effects of entrainment.
2. System mean void fraction invariant with time.
3. Random fluctuations due to the stochastic nature of the two-phase flow process are assumed not to influence the deterministic transients.
4. The spatially averaged evaporator heat flux is time-invariant.
5. Viscous dissipation, longitudinal heat conduction and changes in kinetic energy are neglected.
6. The specific enthalpies and densities of the liquid and vapor are considered to be saturated properties, independent of both axial position and time, and evaluated at the mean system pressure.

Conservation of mass and energy principles

As mentioned earlier, the assumption of the system mean void fraction being invariant with time eliminates the need for the transient form of the momentum principle, although the numerical evaluation of the system mean void fraction requires the steady-state form of the momentum principle. However, when the governing equations for an evaporating flow system are formulated in terms of the system mean void fraction, only the conservation of mass and energy principles are directly involved.

Two-phase region. Using the system mean void fraction model, and incorporating the foregoing simplifications, the conservation of mass principle, simultaneously applied to the liquid and vapor in the two-phase region, can be expressed as

$$\frac{d}{dt}\{[\rho(1-\bar{\alpha}_s)+\rho'\bar{\alpha}_s]A_t\bar{\eta}(t)\} = \bar{m}_t(z,t)_{z=0} - \bar{m}_t^*. \qquad [7]$$

Physically, the expression on the L.H.S. of [7] represents the instantaneous time rate of change of the mass of liquid and vapor within the two-phase region. The first term on the R.H.S. represents the instantaneous rate at which mass enters the two-phase region, and the second term represents the instantaneous rate at which mass leaves the two-phase region *relative* to the transition point boundary.

Similarly, the conservation of energy principle, simultaneously applied to the liquid and vapor in the two-phase region, can be expressed in terms of the system mean void fraction as

$$\frac{d}{dt}\{[\rho h(1-\bar{\alpha}_s)+\rho' h'\bar{\alpha}_s]A_t\bar{\eta}(t)\} = \bar{f}_q P\bar{\eta}(t) + \{[h(1-\bar{x})+h'\bar{x}]\bar{m}_t(z,t)\}_{z=0} - h'\bar{m}_t^*. \qquad [8]$$

where $\bar{f}_q$ represents the spatially averaged evaporator heat flux; defined as

$$\bar{f}_q \equiv \frac{1}{\bar{\eta}(t)} \int_{z=0}^{\bar{\eta}(t)} f_q \, \mathrm{d}z . \qquad [9]$$

The meaning of the L.H.S. of [8] is that it represents the instantaneous time rate of change of the thermal energy of the liquid and vapor within the two-phase region. The first term on the R.H.S. represents the instantaneous rate at which energy is being added to the two-phase region in the form of heat, the second term represents the instantaneous rate at which thermal energy enters the two-phase region by virtue of the inlet mass flowrate, and the last term represents the instantaneous rate at which thermal energy leaves the two-phase region, by virtue of mass leaving, relative to the transition point boundary.

Superheated vapor region. Assuming the vapor density in the superheat region is essentially constant, and that for all practical purposes it can be approximated by that of the saturated vapor, the conservation of the mass principle applied to the superheat region can be expressed by

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\rho' A_t[L - \bar{\eta}(t)]\} = \bar{m}_t^* - \bar{m}_t(z, t)_{z=L} . \qquad [10]$$

Physically, the L.H.S. of [10] represents the time rate of change of the mass in the superheat region, the first term on the R.H.S. represents the mass flowrate entering from the two-phase region relative to the transition point boundary, and the second term represents the mass flowrate of superheated vapor leaving the evaporator.

Transient response of mixture–vapor transition point (effective liquid dry-out point)

Although this particular application has been presented in an earlier paper by Wedekind & Stoecker (1968), it will be outlined again here as one of the four particular applications to be presented, primarily because its inclusion is important to the completeness and symmetry of the presentation.

The transient response of the transition point to a change in the system inlet flowrate can be predicted from the governing differential equations. This will be done in general, and then for the special case where the inlet flowrate can be expressed as an exponential function of time. To confirm the utility of the model, its predictions will then be compared to experimental data.

System equations; arbitrary inlet flowrate. Consistent with the system simplifications stated earlier, [7] and [8] can be combined to yield the following differential equation governing the transient response of the mixture–vapor transition point (effective liquid dry-out point), $\bar{\eta}(t)$, in terms of the system inlet flowrate, $\bar{m}_t(z, t)_{z=0}$:

$$\frac{\mathrm{d}\bar{\eta}(t)}{\mathrm{d}t} + \frac{1}{\tau_e}\bar{\eta}(t) = \frac{(1 - \bar{x}_0)}{\rho(1 - \bar{\alpha}_s)A_t} \bar{m}_t(z, t)_{z=0} , \qquad [11]$$

where τ_e is the system time constant for evaporating flow and defined as

$$\tau_e \equiv \frac{\rho(1 - \bar{\alpha}_s)A_t(h' - h)}{\bar{f}_q P} \qquad [12]$$

and $\bar{x}_0$ represents the inlet flow quality. It should be pointed out that the system time constant for evaporating flow, τ_e, can be thought of as representing the time required to evaporate the liquid which is present within the two-phase region at any instant of time. More detail as to the physical significance of the time constant is given by Wedekind & Stoecker (1968).

The general solution to [11], for an arbitrary inlet flowrate, is given by

$$\bar{\eta}(t)=\frac{(1-\bar{x}_0)}{\rho(1-\bar{\alpha}_s)A_t}\,e^{-(1/\tau_e)t}\int_{\gamma=0}^{t} e^{(1/\tau_e)\gamma}\,\bar{m}_t(z,\gamma)_{z=0}\,d\gamma+\bar{\eta}_i\,e^{-(1/\tau_e)t} \qquad [13]$$

where the initial condition, $\bar{\eta}_i$, is given by

$$\bar{\eta}(t)_{t=0}=\bar{\eta}_i. \qquad [14]$$

Exponential inlet flowrate. As a special case, consider the inlet mass flowrate, $\bar{m}_t(z,t)_{z=0}$, to be adequately represented by the following exponential function of time:

$$\bar{m}_t(z,t)_{z=0}=\bar{m}_{t,f}+(\bar{m}_{t,i}-\bar{m}_{t,f})\,e^{-(1/\tau_m)t}. \qquad [15]$$

where τ_m, is the time constant associated with the inlet flowrate, and the subscripts, i, and, f, refer to the initial and final inlet flowrates respectively.

Substituting [15] into [13], and integrating, the transient response of the effective liquid dry-out point, $\bar{\eta}(t)$, becomes

$$\frac{\bar{\eta}(t)-\bar{\eta}_f}{\bar{\eta}_i-\bar{\eta}_f}=e^{-(1/\tau_e)t}+\frac{1}{\left[\left(\frac{\tau_e}{\tau_m}\right)-1\right]}\{e^{-(1/\tau_e)t}-e^{-(\tau_e/\tau_m)(1/\tau_e)t}\} \qquad [16]$$

where

$$\bar{\eta}_i=\frac{(1-\bar{x}_0)(h'-h)}{\bar{f}_q P}\,\bar{m}_{t,i}$$

and

$$\bar{\eta}_f=\frac{(1-\bar{x}_0)(h'-h)}{\bar{f}_q P}\,\bar{m}_{t,f}. \qquad [17]$$

Note that the situation where $(\tau_e/\tau_m)\rightarrow\infty$, corresponds to the special case of a step change in the inlet flowrate.

Comparison with experimental data. As has already been mentioned, the results of an experimental study of the transient response of the mixture–vapor transition point have been reported earlier in considerable detail. Therefore, a detailed description of the experimental apparatus will not be presented here. However, the evaporator test section was a single, electrically heated, horizontal glass tube approx. 10 m long, with an inside diameter of 1.0 cm. As a means of demonstrating the utility of the model to predict these responses, the experimental results of a decrease in the inlet flowrate for Refrigerant-12 are compared to the model prediction of [16]. This comparison is displayed in figure 2, where considerable agreement is seen to exist. Fujie's (1964) void fraction model was used to evaluate the system mean void fraction. However, as can be seen in the Appendix, other models yield very similar results.

Transient response of the outlet vapor flowrate

System equations; arbitrary inlet flowrate. It is clear from [10] that the outlet vapor flowrate, $\bar{m}_t(z,t)_{z=L}$, is coupled to the response of the mixture–vapor transition point, $\bar{\eta}(t)$. Therefore, combining [7], [10] and [11] yield after rearrangement

$$\bar{m}_t(z,t)_{z=L}=\left\{1-\left[1-\left(\frac{\rho'}{\rho}\right)\right](1-\bar{x}_0)\right\}\bar{m}_t(z,t)_{z=0}+\left[1-\left(\frac{\rho'}{\rho}\right)\right]\frac{\bar{f}_q P}{(h'-h)}\,\bar{\eta}(t). \qquad [18]$$

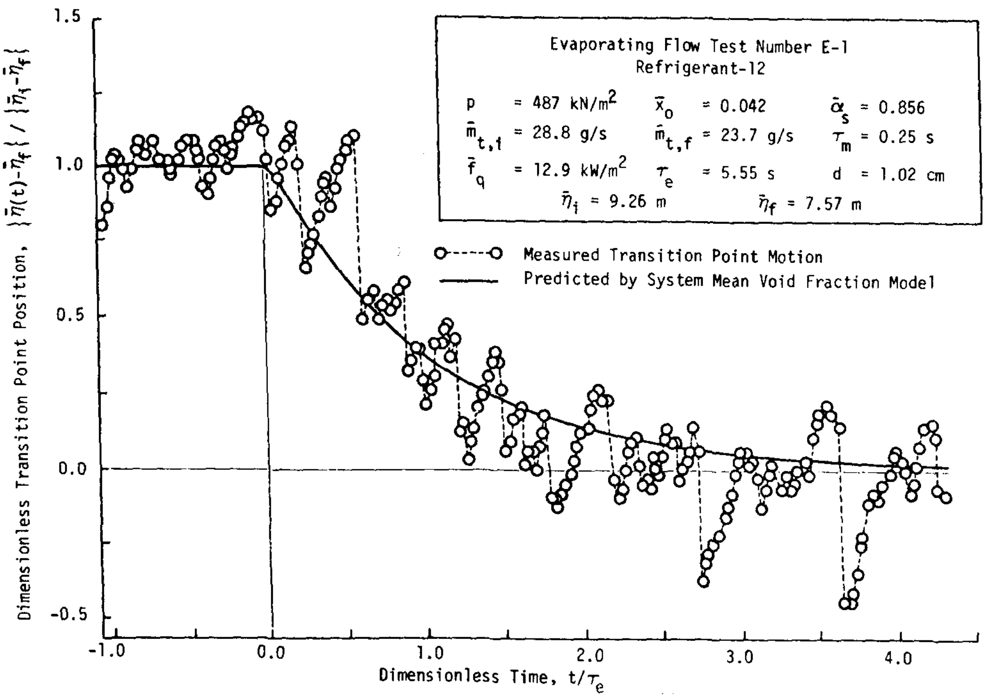

Figure 2. Response of mixture–vapour transition point after a decrease in inlet flowrate.

A general solution of the response of the outlet vapor flowrate to an arbitrary inlet flowrate can thus be obtained by combining the general response of the transition point, $\bar{\eta}(t)$, as expressed by [13], with the above expression.

Exponential inlet flowrate. Again as a special case, consider the inlet mass flowrate, $\bar{m}_t(z,t)_{z=0}$, to be an exponential function of time. Thus, [16] will represent the transient response of the transition point to such an inlet flowrate. Substituting [15]–[17] into [18], yields after rearrangement

$$\frac{\bar{m}_t(z,t)_{z=L}-\bar{m}_{t,f}}{\bar{m}_{t,i}-\bar{m}_{t,f}}=e^{-(\tau_e/\tau_m)(1/\tau_e)t}+\frac{\left(\frac{\tau_e}{\tau_m}\right)(1-\bar{x}_0)\left[1-\left(\frac{\rho'}{\rho}\right)\right]}{\left[\left(\frac{\tau_e}{\tau_m}\right)-1\right]}\{e^{-(1/\tau_e)t}-e^{-(\tau_e/\tau_m)(1/\tau_e)t}\}. \qquad [19]$$

The above expression simplifies when, $(\tau_e/\tau_m)\to\infty$, which corresponds to a step change in the inlet flowrate.

Comparison with experimental data. The experimental apparatus used to obtain the transient response of the outlet vapor flowrate to a change in the inlet flowrate was completely different from the apparatus used for the transition point response. A detailed description of this particular apparatus is described in previous work by Wedekind (1971). However, the specific evaporator test section consisted of nine, 1.0 m horizontal glass tubes arranged in a vertical serpentine configuration, electrically heated, having a total length of approx. 9 m, with an inside diameter of 0.84 cm. Typical results are depicted in figure 3 for Refrigerant-12. The agreement between the experimental data and the response predicted by the system mean void fraction model are quite reasonable.

Further insight into the physical mechanisms behind the response characteristics of the outlet vapor flowrate to a change in the evaporator inlet flowrate can be obtained by examining the conservation of mass equations for both the two-phase and the superheated vapor region. Combining [7] and [10] in such a manner as to eliminate the mass flowrate, $\bar{m}_t^*$, leaving the

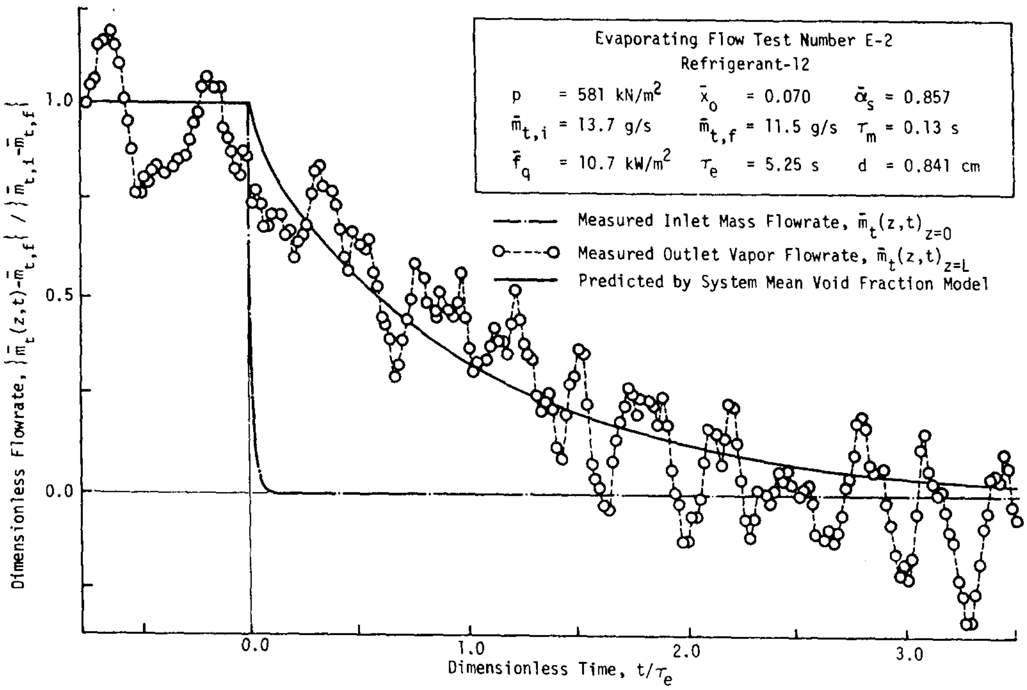

Figure 3. Response of outlet vapour flowrate after a decrease in inlet flowrate.

two-phase region relative to the transition point, boundary, yields

$$\bar{m}_t(z,t)_{z=L} = \bar{m}_t(z,t)_{z=0} - (\rho - \rho')(1 - \bar{\alpha}_s)A_t \frac{d\bar{\eta}(t)}{dt}. \qquad [20]$$

It can be seen from the above expression that the outlet flowrate of super-heated vapor, $\bar{m}_t(z,t)_{z=L}$, is equal to the evaporator inlet flowrate, $\bar{m}_t(z,t)_{z=0}$, minus the net rate at which mass is stored in the evaporator as a result of liquid displacing vapor. With insight from [11], and the fact that $(\rho > \rho')$, the net storage rate in the evaporator is seen to be negative if the inlet flowrate decreases from a steady state configuration. This storage term causes a momentary delay to the decrease in the outlet flowrate of superheated vapor (in effect, a form of a momentary excess liquid hold-up within the evaporator).

The inverse takes place if the inlet flowrate increases from a steady state configuration. However, once the mixture–vapor transition point, $\bar{\eta}(t)$, has reached a new steady state position, the inlet and outlet flowrates will obviously be equal.

APPLICATION TO CONDENSING FLOW TRANSIENTS

This section of the paper is directed at the application of the system mean void fraction model to predict the transient responses of two-phase condensing flow systems. The two specific system responses to be considered are those of the effective point of complete condensation, and those of the outlet flowrate of subcooled liquid; where from a similarity with the evaporator case, the second is dependent upon the first.

Introduction

A schematic of the condensing flow system is shown in figure 4. In addition to the system mean void fraction being invariant with time, analogous to the evaporator, it will be assumed that there is some effective position in the condenser where the last of the vapor is condensed. This position may not be as clearly defined as for the evaporator, but such an effective position

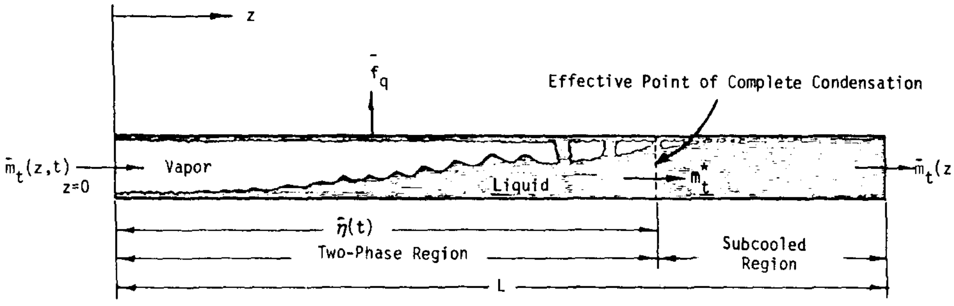

Figure 4. Schematic of horizontal condensing flow system.

can be conceived. This type of effective point of complete condensation or mixture–liquid transition point has been observed by Soliman & Berenson (1970). This effective transition point again will also be designated by the symbol, $\bar{\eta}(t)$. Therefore, [4] is a valid expression for the system mean void fraction. Also, the same simplifying assumptions used in the evaporating flow system model are used for the condensing flow system.

Conservation of mass and energy principles

As with the evaporating flow system, the assumption that the system mean void fraction is independent of time eliminates the need for the transient form of the momentum principle. Therefore, only the conservation of mass and energy principles are directly involved.

Two-phase region. Utilizing the system mean void fraction model, and incorporating the forestated simplifications, the conservation of mass principle, simultaneously applied to the liquid and vapor in the two phase region, will be identical to that of [7] for evaporating flow.

The conservation of energy principle, simultaneously applied to the liquid and vapor in the two-phase region, can be expressed in terms of the system mean void fraction as

$$\frac{\mathrm{d}}{\mathrm{d}t}\{[\rho h(1-\bar{\alpha}_s)+\rho' h'\bar{\alpha}_s]A_t\bar{\eta}(t)\} = -\bar{f}_q P\bar{\eta}(t) + \{[h(1-\bar{x})+h'\bar{x}]\bar{m}_t(z,t)\}_{z=0} - h\bar{m}_t^* \quad [21]$$

where $\bar{f}_q$ represents the average condenser heat flux; defined by [9]. The physical meaning of each term in the above equation is term by term identical with that of [8], with the exception of the direction of heat transfer which is out of the two-phase region for condensing flow.

Subcooled liquid region. The conservation of mass principle applied to the subcooled liquid region can be expressed as

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\rho A_t[L-\bar{\eta}(t)]\} = \bar{m}_t^* - \bar{m}_t(z,t)_{z=L}. \quad [22]$$

With the exception of the fact that the fluid in the above equation is liquid, the physical meaning of each term is identical with that of [10].

Transient response of mixture–liquid transition point (effective point of complete condensation)

System equations; arbitrary inlet flowrate. Consistent with the system simplifications stated earlier, [7] and [21] can be combined to yield the differential equation governing the transient response of the mixture–liquid transition point (effective point of complete condensation), $\bar{\eta}(t)$, in terms of the system inlet flowrate, $\bar{m}_t(z,t)_{z=0}$.

$$\frac{\mathrm{d}\bar{\eta}(t)}{\mathrm{d}t} + \frac{1}{\tau_c}\bar{\eta}(t) = \frac{\bar{x}_0}{\rho'\bar{\alpha}_s A_t}\bar{m}_t(z,t)_{z=0} \quad [23]$$

where τ_c is the time constant for the condensing flow system; defined as

$$\tau_c \equiv \frac{\rho' \bar{\alpha}_s A_t (h' - h)}{\bar{f}_q P} \quad [24]$$

and $\bar{x}_0$ represents the inlet flow quality. For condensing flow, the system time constant, τ_c, can be thought of as representing the time required to condense the vapor which is present within the two-phase region at any instant of time.

Equation [23] has a general solution

$$\bar{\eta}(t) = \frac{\bar{x}_0}{\rho' \bar{\alpha}_s A_t} e^{-(1/\tau_c)t} \int_{\gamma=0}^{t} e^{(1/\tau_c)\gamma} \bar{m}_t(z, \gamma)_{z=0} \, d\gamma + \bar{\eta}_i e^{-(1/\tau_c)t} \quad [25]$$

where the initial condition, $\bar{\eta}_i$, is given by [14].

Exponential inlet flowrate. As a special case, consider the inlet mass flowrate, $\bar{m}_t(z, t)_{z=0}$, to be adequately represented by an exponential function of time. Therefore, substituting [15] into [25] and integrating, the transient response of the effective point of complete condensation, $\bar{\eta}(t)$, becomes

$$\frac{\bar{\eta}(t) - \bar{\eta}_f}{\bar{\eta}_i - \bar{\eta}_f} = e^{-(1/\tau_c)t} + \frac{1}{\left[\left(\frac{\tau_c}{\tau_m}\right) - 1\right]} \{e^{-(1/\tau_c)t} - e^{-(\tau_c/\tau_m)(1/\tau_c)t}\} \quad [26]$$

where

$$\bar{\eta}_i = \frac{\bar{x}_0 (h' - h)}{\bar{f}_q P} \bar{m}_{t,f}$$

and

$$\bar{\eta}_f = \frac{\bar{x}_0 (h' - h)}{\bar{f}_q P} \bar{m}_{t,f} . \quad [27]$$

Note that as with the evaporating flow, the situation where $(\tau_c/\tau_m) \to \infty$ corresponds to the special case of a step change in the inlet flowrate.

No direct experimental data exists at this time for the transient response of the effective point of complete condensation. However, because the response of this transition point, $\bar{\eta}(t)$, is directly coupled to the transient response of the outlet liquid flowrate, the foregoing model can be indirectly verified by experimentally measuring the response of the outlet liquid flowrate.

Transient response of the outlet liquid flowrate

System equation; arbitrary inlet flowrate. As was the case for evaporatiog flow, the transient response of the outlet liquid flowrate from a condenser, $\bar{m}_t(z, t)_{z=L}$, is directly coupled to the response of the transition point, $\bar{\eta}(t)$. Therefore, combining [7], [22] and [23] yield after rearrangement

$$\bar{m}_t(z, t)_{z=L} = \left\{1 + \left[\left(\frac{\rho}{\rho'}\right) - 1\right] \bar{x}_0\right\} \bar{m}_t(z, t)_{z=0} - \left[\left(\frac{\rho}{\rho'}\right) - 1\right] \frac{\bar{f}_q P}{(h' - h)} \bar{\eta}(t) . \quad [28]$$

In a manner similar to that for evaporating flow, a general solution of the response of the outlet liquid flowrate to an arbitrary inlet flowrate can be obtained by combining the general response of the transition point, $\bar{\eta}(t)$, as expressed by [25], with the above equation.

Exponential inlet flowrate. The transient response of the condenser outlet liquid flowrate, due to an exponential inlet flowrate can be obtained by substituting [15], [26] and [27] into [28] and rearranging; thus

$$\frac{\bar{m}_t(z,t)_{z=L} - \bar{m}_{t,f}}{\bar{m}_{t,i} - \bar{m}_{t,f}} = e^{-(\tau_c/\tau_m)(1/\tau_c)t} + \frac{\left(\frac{\tau_c}{\tau_m}\right)\left[\left(\frac{\rho}{\rho'}\right) - 1\right]\bar{x}_0}{\left[\left(\frac{\tau_c}{\tau_m}\right) - 1\right]}\{e^{-(\tau_c/\tau_m)(1/\tau_c)t} - e^{-(1/\tau_c)t}\}. \quad [29]$$

Again, the above expression reduces to the special case of a step change in inlet flowrate as $(\tau_c/\tau_m) \to \infty$.

Comparison with experimental data. The ability of the system mean void fraction model to predict the transient response of the condenser outlet liquid flowrate has been investigated experimentally by Wedekind & Bhatt (1976), where a detailed description of the experimental apparatus is given. However, the condenser test section consisted of a single horizontal water-cooled copper concentric tube heat exchanger approx. 5 m long. The condensing flow was inside the inner tube which had an inside diameter of 0.8 cm. The experimental data shown here is for Refrigerant-12. Responses for both an increase, as well as a decrease in inlet flowrate will be presented.

The response of the outlet liquid flowrate due to an exponential increase in the inlet flowrate is presented in figure 5. The agreement is quite reasonable, and the response is very interesting, especially the overshoot of the outlet flowrate immediately after the inlet flowrate change. Zivi's (1964) void fraction model was used to evaluate the system mean void fraction. The response of the outlet liquid flowrate due to an exponential decrease in the inlet flowrate, as depicted in figure 6, again manifests a sizeable overshoot. In fact, for a short period of time, the outlet liquid flowrate actually reverses its direction, flowing back into the condenser.

As with evaporating flow, further insight into the physical mechanisms behind the overshoot characteristics of the outlet liquid flowrate to a change in the condenser inlet flowrate can be obtained by examining the conservation of mass equations for both the two-phase and the subcooled liquid region. Combining[7] and [22] in such a manner as to eliminate the mass flow-

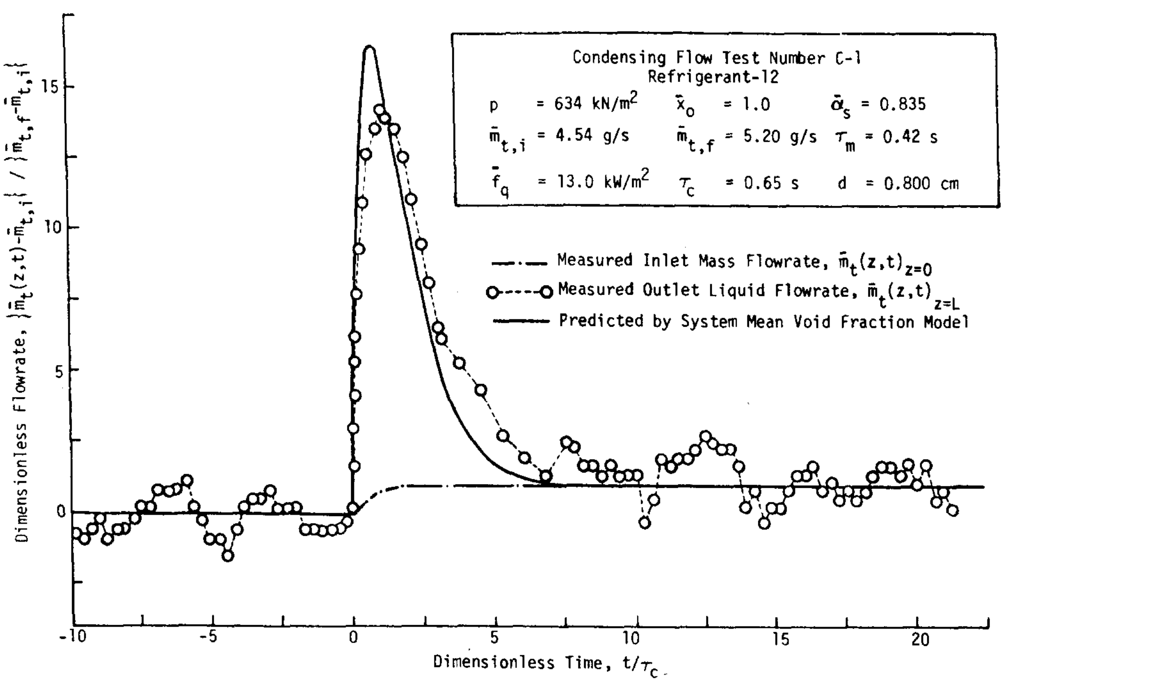

Figure 5. Response of outlet liquid flowrate after an increase in outlet flowrate.

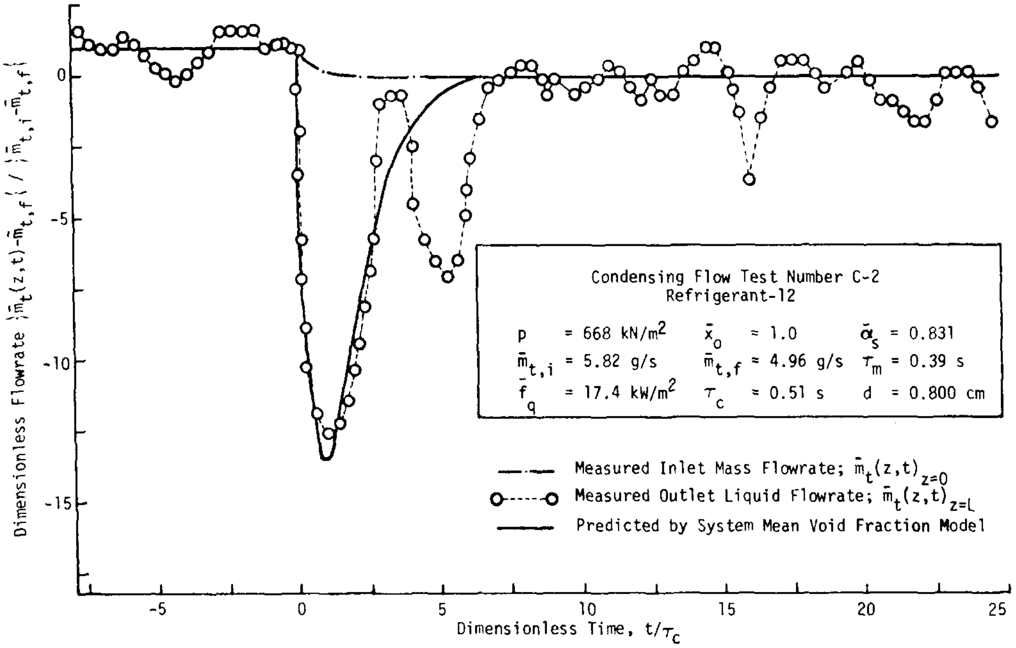

Figure 6. Response of outlet liquid flowrate after a decrease in inlet flowrate.

rate, $\bar{m}_t^*$, leaving the two-phase region relative to the transition point boundary, yields

$$\bar{m}_t(z, t)_{z=L} = \bar{m}_t(z, t)_{z=0} - (\rho' - \rho)\bar{\alpha}_s A_t \frac{d\bar{\eta}(t)}{dt}. \qquad [30]$$

It can be seen from the above expression that the outlet flowrate of subcooled liquid, $\bar{m}_t(z, t)_{z=L}$, is equal to the condenser inlet flowrate, $\bar{m}_t(z, t)_{z=0}$, minus the net rate at which mass is stored in the condenser as a result of vapor displacing liquid. With insight from [23], and the fact that $(\rho' < \rho)$, the net storage rate in the condenser is seen to be positive if the inlet flowrate decreases from a steady state configuration. This causes the outlet liquid flowrate to be momentarily smaller than the inlet flowrate (in effect, a form of a momentary liquid shortage in the condenser). It should be pointed out that the magnitude of this momentary overshoot is directly proportional to the liquid/vapor density difference.

The inverse takes place if the inlet flowrate increases from a steady state configuration. Once the mixture–liquid transition point, $\bar{\eta}(t)$, has reached a new steady state position however, the inlet and outlet flowrates will obviously be equal.

SUMMARY AND CONCLUSIONS

The *system mean void fraction model* converts the two-phase evaporating or condensing flow system into a type of lumped parameter system, generally yielding simple, closed form solutions in terms of the important system parameters. The major assumption in the model is that the system mean void fraction is time-invariant. This assumption uncouples the problem from the transient form of the momentum principle.

The four specific transient responses considered in this paper are for tube-type evaporators or condensers, where complete vaporization or condensation takes place. All of the responses are initiated by changes in the inlet flowrate of the evaporating or condensing fluid. For evaporating flows, the specific transients are the effective liquid dry-out point, and the outlet flowrate of superheated vapor. Similarly, for condensing flows, they are the effective point of complete condensation, and the outlet flowrate of subcooled liquid.

The ability of the model to accurately predict these transient responses has been demonstrated by experimental data. It is important to point out however, that the model as presented here cannot predict the random fluctuations which are inherent in the two-phase flow. The model seems to have the capability of accurately handling both step and slowly changing inlet flowrates.

It is important to recognize that the system mean void fraction model does not contain any empirical constants as such. Also, in the light of the various applications presented in this paper, the model is seen to embody a certain analytical simplicity, and yet maintain the capability of accurately predicting various transient phenomena associated with both evaporating and condensing flows. These predictions include the influence of evaporator and condenser geometry, heat flux, flowrate, fluid properties and inlet conditions. This is borne out by experimental data, including the fact that, as was pointed out earlier, the data have been obtained from three distinctly different experimental apparatuses.

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REFERENCES

CHATO, J. C. 1962 Laminar condensation. *ASHRAE Jl* **4**, 52–60.

COLLIER, J. G. 1972 *Convective Boiling and Condensation.* McGraw-Hill, New York.

DOROSHCHUK, V. Ye & FRID, F. P. 1969 The critical heat flux for water flowing through round tubes. *Problems of Heat Transfer and Hydraulics of Two-Phase Media* (Edited by Kutateladze, S. S.) Chapter 2, pp. 38–40. Pergamon Press, Oxford.

FUJIE, H. 1964 A relation between steam quality and void fraction in two-phase flow. *A.I.Ch.E.Jl* **10**, 227–232.

GONZALEZ-SANTALO, J. M. & LAHEY, R. T. Jr. 1973 An exact solution for flow transients in two-phase systems by the method of characteristics. *J. Heat Transfer* Ser. C. **95**, 470–476.

HANCOX, W. T. & NICOLL, W. B. 1971 A general technique for the prediction of void distributions in non-steady two-phase forced convection. *Int. J. Heat Mass Transfer* **14**, 1377–1394.

HUDSON, L., ATIT, K. M. & BANKOFF, S. G. 1964 Response of a boiling channel to power or inlet flow modulation. *Chem. Engng Sci.* **19**, 387–401.

HUFSCHMIDT, W. 1960 Die eigenschaften von rippenrohrluftkuhlern im arbeitsbereich der klimaanlagen. *ForschBer. Landes NRhein-Westf.* No. 889.

JONES, O. C. & ZUBER, N. 1975 The interrelation between void fraction fluctuations and flow patterns in two-phase flow. *Int. J. Multiphase Flow* **2**, 273–306.

LEVY, S. 1960 Steam-slip-theoretical prediction from momentum model. *J. Heat Transfer* **82**, 113–124.

RUFER, C. E. & KEZIOS, S. P. 1966 Analysis of two-phase one-component stratified flow with condensation. *J. Heat Transfer* **88**, 265–275.

SACKS, P. S. 1975 Measured characteristics of adiabatic and condensing single-component two-phase flow of refrigerant in a 0.377 in. diameter horizontal tube. ASME paper No. 75 WA/HT-24, Winter Annual Meeting, Houston, Texas, pp. 1–12.

SCHOENBERG, A. A. 1966 Mathematical model with experimental verification for the dynamic behavior of a single-tube condenser, NASA TN D-3453, pp. 1–51.

SOLIMAN, M. & BERENSON, P. J. 1970 Flow stability and gravitational effect in condenser tubes. *Proceeding from the Fourth International Heat Transfer Conference*, Paris, France, Vol. VI, CS 1.8, 1–14.

ST. PIERRE, C. C. 1965 Frequence-response analysis of steam voids to sinusoidal power modulation in a thin-walled boiling water coolant channel. ANL-7041, pp. 1–46.

STEEN, P. A. & WALLIS, G. B. 1964 AEC report NYO-3114-2.

VILD, T. J., SCHUBERT, F. H. & SNOKE, D. R. 1968 A system to demonstrate the zero gravity performance of an organic rankine-cycle. Inter-society Energy Conversion Engineering Conference, Boulder, Colorado, pp. 376–388.

WALLIS, G. B. 1969 *One-Dimensional Two-Phase-Flow*. McGraw-Hill, New York.

WEDEKIND, G. L. 1965 Transient response of the mixture–vapor transition point in two-phase horizontal evaporating flow. Ph.D. Thesis, University of Illinois, pp. 63–70.

WEDEKIND, G. L. & STOECKER, W. F. 1968 Theoretical model for predicting the transient response of the mixture–vapor transition point in horizontal evaporating flow. *J. Heat Transfer* **90**, 165–174.

WEDEKIND, G. L. 1971 An experimental investigation into the oscillatory motion of the mixture–vapor transition point in horizontal evaporating flow. *J. Heat Transfer* **93**, 47–54.

WEDEKIND, G. L. & BECK, B. T. 1974 Theoretical model of the mixture–vapor transition point oscillations associated with two-phase evaporating flow instabilities. *J. Heat Transfer* **96**, 138–144.

WEDEKIND, G. L. & BHATT, B. L. 1976 An experimental and theoretical investigation into thermally governed transient flow surges in two-phase condensing flow. *Proceedings of the Two-Phase Flow and Heat Transfer Symposium–Workshop*, Ft. Lauderdale, Florida, October. *J. Heat Transfer.* In press.

ZIVI, S. M. 1964 Estimation of steady-state steam void fraction by means of the principle of minimum entropy production. *J. Heat Transfer* **86**, 247–252.

ZUBER, N. & STAUB, F. W. 1966 The propagation and the wave form of the vapor volumetric concentration in boiling, forced convection systems under oscillatory conditions, *Int. J. Heat Mass Transfer* **9**, 871–894.

APPENDIX

Evaluation of system mean void fraction

Evaporating flow. For steady evaporating flow conditions, with a uniform heat flux,† the following relationship between flow quality, $\bar{x}$, and the dimensionless variable, ξ, exists:

$$\bar{x} = \bar{x}_0 + (1 - \bar{x}_0)\xi. \tag{31}$$

Substitution of this relationship into [4] for the system mean void fraction yields

$$\bar{\alpha}_s = \int_{\xi=0}^{1} \bar{\alpha}_a(\xi)\,\mathrm{d}\xi = \frac{1}{(1-\bar{x}_0)} \int_{\bar{x}=\bar{x}_0}^{1} \bar{\alpha}_a(\bar{x})\,\mathrm{d}\bar{x}. \tag{32}$$

For steady flow conditions, Levy (1960), Fujie (1964) and Zivi (1964) have proposed theoretical models relating void fraction and flow quality. Figure 7 displays a comparison of these three models with Hufschmidt's (1960) experimental data for horizontal flow of Refrigerant-12. While Fujie's and Levy's models appear to better represent Hufschmidt's experimental data, both models require numerical integration to obtain the system mean void fraction. However, Zivi's model can be used to express the system mean void fraction in a simple closed form; thus

$$\bar{\alpha}_s = \frac{1}{(1-c)} + \frac{c}{(1-\bar{x}_0)(1-c)^2} \ln\{c + (1-c)\bar{x}_0\} \tag{33}$$

where the constant, c, is defined by $c \equiv \{\rho'/\rho\}^{2/3}$.

†If the heat flux is not uniform, although it would not be as accurate, an estimate could still be obtained by using this relationship.

A comparison of the calculated system mean void fraction using the above three models for the evaporating flow tests which are presented in this paper are shown below:

Test number	Levy's model	Fujie's model	Zivi's model
E-1	0.863	0.856	0.881
E-2	0.869	0.857	0.884

Condensing flow. For steady condensing flow conditions,† the local flow quality may be expressed as

$$\bar{x} = \bar{x}_0(1 - \xi). \qquad [34]$$

Substituting this relationship into [4] yields the following expression for the system mean void fraction:

$$\bar{\alpha}_s = \int_{\xi=0}^{1} \bar{\alpha}_a(\xi)\,d\xi = \frac{1}{\bar{x}_0}\int_{\bar{x}=0}^{\bar{x}_0} \bar{\alpha}_a(\bar{x})\,d\bar{x} \qquad [35]$$

It will be assumed† that the same models developed for evaporating flows are also valid for condensing flows. Therefore, a comparison of the system mean void fraction for the condensing flow data presented in this paper are shown below:

Test number	Levy's model	Fujie's model	Zivi's model
C-1	0.825	0.819	0.835
C-2	0.822	0.816	0.831

Chato (1962) and Rufer & Kezios (1966) studied stratified flow condensation inside of a circular tube. Chato suggests an average value of 120° for the angle whose vertex is at the center of the tube, and which is subtended by the condensate. Similar results can be obtained from the work

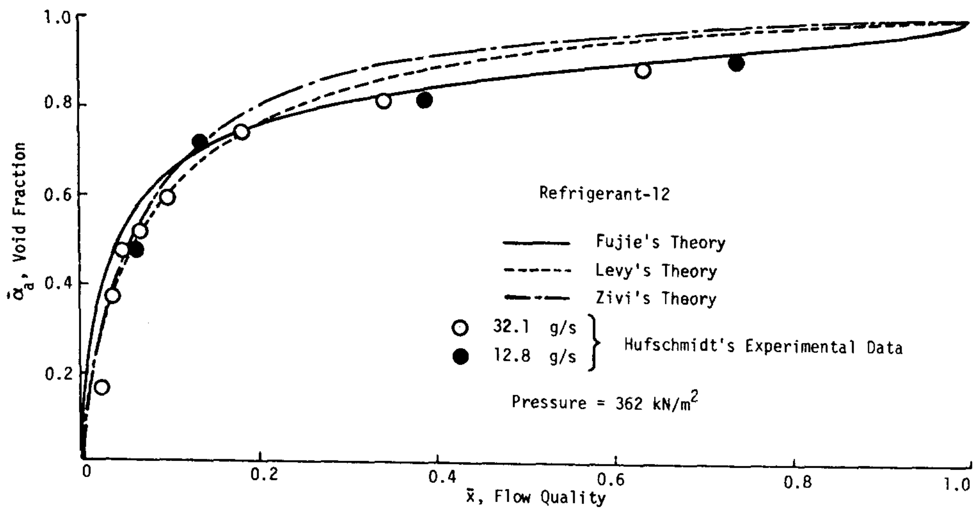

Figure 7. Comparison of several void fraction vs flow quality models.

†This can be substantiated by comparison with the experimental data of Sacks (1975) for adiabatic and condensing flows.

of Rufer and Kezios. The system mean void fraction, based on this angle of 120°, is 0.805, which is in reasonable agreement with the previous values. Therefore, for simplicity, it is convenient to use Zivi's model for estimating the system mean void fraction, thus for condensing flow

$$\bar{\alpha}_s = \frac{1}{(1-c)} + \frac{c}{(1-c)^2\bar{x}_0} \ln\left\{\frac{c}{(1-c)\bar{x}_0 + c}\right\} \qquad [36]$$

where the constant, c, is defined by $c \equiv \{\rho'/\rho\}^{2/3}$.